

# Physics Notes Class 11 CHAPTER MECHANICAL PROPERTIES OF FLUIDS part 2

## Viscosity

The property of a fluid by virtue of which an internal frictional force acts between its different layers which opposes their relative motion is called viscosity.

These internal frictional force is called viscous force.

Viscous forces are intermolecular forces acting between the molecules of different layers of liquid moving with different velocities.

$$\text{Viscous force } (F) = -\eta A \frac{dv}{dx}$$
$$\eta = -\frac{F}{A \left( \frac{dv}{dx} \right)}$$

where,  $(dv/dx)$  = rate of change of velocity with distance called velocity gradient,  $A$  = area of cross-section and  $\eta$  = coefficient of viscosity.

SI unit of  $\eta$  is  $\text{Nsm}^{-2}$  or pascal-second or decapoise. Its dimensional formula is  $[\text{ML}^{-1}\text{T}^{-1}]$ .

The knowledge of the coefficient of viscosity of different oils and its variation with temperature helps us to select a suitable lubricant for a given machine.

Viscosity is due to transport of momentum. The value of viscosity (and compressibility) for ideal liquid is zero.

The viscosity of air and of some liquids is utilised for damping the moving parts of some instruments.

The knowledge of viscosity of some organic liquids is used in determining the molecular weight and shape of large organic molecules like proteins and cellulose.

## Variation of Viscosity

The viscosity of liquids decreases with increase in temperature

$$\eta_t = \frac{\eta_0}{(1 + \alpha t + \beta t^2)}$$

where,  $\eta_0$  and  $\eta_t$  is are coefficient of viscosities at  $0^\circ\text{C}$   $t^\circ\text{C}$ ,  $\alpha$  and  $\beta$  are constants.

The viscosity of gases increases with increase in temperatures as

$$\eta \propto \sqrt{T}$$

The viscosity of liquids increases with increase in pressure but the viscosity of water decreases with increase in pressure.

The viscosity of gases do not changes with pressure.

### Poiseuille's Formula

The rate of flow ( $v$ ) of liquid through a horizontal pipe for steady flow is given by

$$v = \frac{\pi p r^4}{8 \eta l}$$

where,  $p$  = pressure difference across the two ends of the tube.  $r$  = radius of the tube,  $\eta$  = coefficient of viscosity and  $l$  = length of the tube.

### The Rate of Flow of Liquid

Rate of flow of liquid through a tube is given by

$$v = (P/R)$$

where,  $R = (8 \eta l / \pi r^4)$ , called liquid resistance and  $p$  = liquid pressure.

#### (i) When two tubes are connected in series

- Resultant pressure difference  $p = p_1 + p_2$
- Rate of flow of liquid ( $v$ ) is same through both tubes.
- Equivalent liquid resistance,  $R = R_1 + R_2$

#### (ii) When two tubes are connected in parallel

1. Pressure difference ( $p$ ) is same across both tubes.
2. Rate of flow of liquid  $v = v_1 + v_2$
3. Equivalent liquid resistance  $(1/R) = (1/R_1) + (1/R_2)$

## Stoke's Law

When a small spherical body falls in a long liquid column, then after sometime it falls with a constant velocity, called terminal velocity. When a small spherical body falls in a liquid column with terminal velocity then viscous force acting on it is

$$F = 6\pi\eta rv$$

where,  $r$  = radius of the body,  $V$  = terminal velocity and  $\eta$  = coefficient of viscosity.

This is called **Stoke's law**.

$$\text{Terminal velocity } v = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$

where,

- $\rho$  = density of body,
  - $\sigma$  = density of liquid,
  - $\eta$  = coefficient of viscosity of liquid and,
  - $g$  = acceleration due to gravity
1. If  $\rho > \rho_0$ , the body falls downwards.
  2. If  $\rho < \rho_0$ , the body moves upwards with the constant velocity.
  3. If  $\rho_0 \ll \rho$ ,  $v = (2r^2\rho g/9\eta)$

## Importance of Stoke's Law

1. This law is used in the determination of electronic charge by Millikan in his oil drop experiment.
2. This law helps a man coming down with the help of parachute.
3. This law account for the formation of clouds.

## Flow of Liquid

1. **Streamline Flow** The flow of liquid in which each of its particle follows the same path as followed by the proceeding particles, is called streamline flow.
2. **Laminar Flow** The steady flow of liquid over a horizontal surface in the form of layers of different velocities, is called laminar flow.
3. **Turbulent Flow** The flow of liquid with a velocity greater than its critical velocity is disordered and called turbulent flow.

## Critical Velocity

The critical velocity is that velocity of liquid flow, below which its fl is streamlined and above which it becomes turbulent.

$$\text{Critical velocity } v_c = (k_\eta/r\rho)$$

where,

- $K$  = Reynold's number,
- $\eta$  = coefficient of viscosity of liquid
- $r$  = radius of capillary tube and  $\rho$  = density of the liquid.

### Reynold's Number

Reynold's number is a pure number and it is equal to the ratio of inertial force per unit area to the viscous force per unit area for flowing fluid.

$$\text{Reynold number } K = \frac{v_c \rho r}{\eta}$$

where,  $\rho$  = density of the liquid and  $v_c$  = critical velocity.

For pure water flowing in a cylindrical pipe,  $K$  is about 1000.

When  $0 < K < 2000$ , the flow of liquid is streamlined.

When  $2000 < K < 3000$ , the flow of liquid is variable betw streamlined and turbulent.

When  $K > 3000$ , the flow of liquid is turbulent.

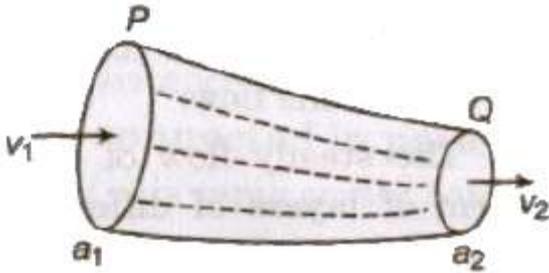
It has no unit and dimension.

### Equation of Continuity

If a liquid is flowing in streamline flow in a pipe of non-unif cross-section area, then rate of flow of liquid across any cross-sec remains constant.

$$a_1 v_1 = a_2 v_2 \quad a v = \text{constant}$$

The velocity of liquid is slower where area of cross-section is larger faster where area of cross-section is smaller.



The falling stream of water becomes narrower, as the velocity of the stream of water increases and therefore its area of cross-section decreases.

### Energy of a Liquid

A liquid in motion possesses three types of energy

(i) Pressure Energy Pressure energy per unit mass =  $p/\rho$

where,

$p$  = pressure of the liquid and  $\rho$  = density of the liquid.

Pressure energy per unit volume =  $p$

(ii) Kinetic Energy

- Kinetic energy per unit mass =  $(1/2)v^2$
- Kinetic energy per unit volume =  $1/2\rho v^2$

(iii) Potential Energy

- Potential energy per unit mass =  $gh$
- Potential energy per unit volume =  $\rho gh$

### Bernoulli's Theorem

If an ideal liquid is flowing in streamlined flow then total energy, i.e., sum of pressure energy, kinetic energy and potential energy per unit volume of the liquid remains constant at every cross-section of the tube.

#### Mathematically

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

It can be expressed as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

where,  $(p/\rho g)$  = pressure head,  $(v^2/2g)$  = velocity head and  $h$  = gravitational head.

For horizontal flow of liquid,

$$p + \frac{1}{2} \rho v^2 = \text{constant}$$

- where,  $p$  is called static pressure and  $(1/2 \rho v^2)$  is called dynamic pressure.
- Therefore in horizontal flow of liquid, if  $p$  increases,  $v$  decreases and vice-versa.
- The theorem is applicable to ideal liquid, i.e., a liquid which is non-viscous incompressible and irrotational.

### Applications of Bernoulli's Theorem

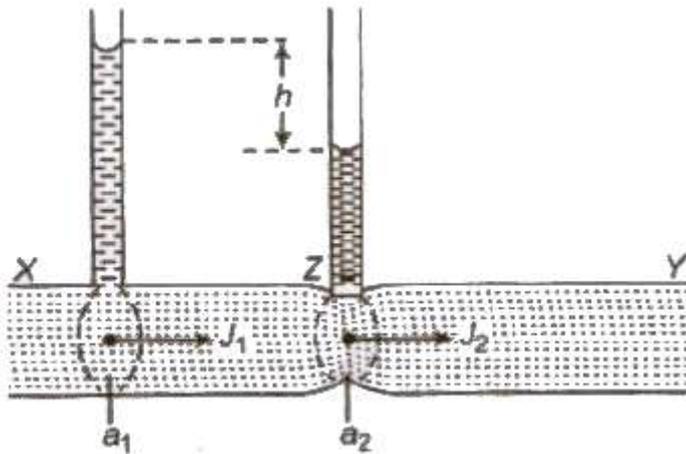
1. The action of carburetor, paintgun, scent sprayer atomiser insect sprayer is based on Bernoulli's theorem.
2. The action of Bunsen's burner, gas burner, oil stove exhaust pump is also based on Bernoulli's theorem.
3. Motion of a spinning ball (Magnus effect) is based on Bernoulli theorem.
4. Blowing of roofs by wind storms, attraction between two close parallel moving boats, fluttering of a flag etc are also based Bernoulli's theorem.

### Venturimeter

It is a device used for measuring the rate of flow of liquid in pipes. Its working is based on Bernoulli's theorem.

Rate of flow of liquid,

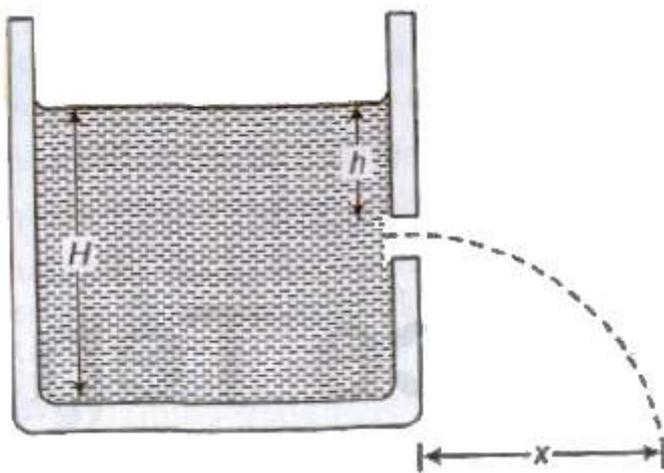
$$v = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$



where,  $a_1$  and  $a_2$  are area of cross-sections of tube at bra and narrower part and  $h$  is difference of liquid columns in ver tubes.

### Torricelli's Theorem

Velocity of efflux (the velocity with which the liquid flows out orifice or narrow hole) is equal to the velocity acquired by a falling body through the same vertical distance equal to the dep orifice below the free surface of liquid.



Velocity of efflux,  $v = \sqrt{2gh}$

where,  $h$  = depth of orifice below the free surface of liquid.

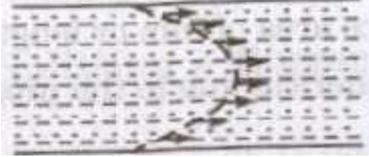
Horizontal range,  $S = \sqrt{4h(H - h)}$

where,  $H$  = height of liquid column.

Horizontal range is maximum, equal to height of the liquid column  $H$ , when orifice is at half of the height of liquid column.

## Important Points

- In a pipe the inner layer (central layer) have maximum velocity and the layer in contact with pipe have least velocity.
- Velocity profile of liquid flow in a pipe is parabolic.



- Solid friction is independent of area of surfaces in contact while viscous force depends on area of liquid layers.
- A lubricant is chosen according to the nature of machinery. In heavy machines lubricating oils of high viscosity are used and in light machines low viscosity oils are used.
- The cause of viscosity in liquids is the cohesive forces among their molecules while cause of viscosity in gases is diffusion.